On the automorphisms of numerical power semigroups

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Abstract. This talk reports new results on the automorphism groups of infinitary power semigroups of numerical semigroups, that is, cofinite subsemigroups of the non-negative integers under addition, extending prior research limited to finitary (finite-subset-only) power structures.

Let S be an additively written semigroup. Denote by $\mathcal{P}(S)$ the semigroup consisting of all non-empty subsets of S endowed with the operation of setwise addition defined by

$$X + Y := \{x + y \colon x \in X,\, y \in Y\}, \qquad \text{for all } X, Y \in \mathcal{P}(S).$$

We call $\mathcal{P}(S)$ the large power semigroup of S. When S is a monoid with neutral element 0_S, the family $\mathcal{P}_0(S)$ of all subsets of S containing 0_S is a submonoid of $\mathcal{P}(S)$, whose identity element is the singleton $\{0_S\}$; we call $\mathcal{P}_0(S)$ the reduced large power monoid of S.

We prove that the automorphism groups of the reduced large power monoid of a numerical monoid and the large power semigroup of a numerical semigroup are both trivial. A key technical tool is the T-conjugacy quotient, which links the automorphism group of $\mathcal{P}(S)$ when S is numerical to the automorphism group of $\mathcal{P}_0(\mathbb{N})$, where $\mathbb{N}$ is the monoid of non-negative integers under addition.

About the speaker. Kerou Wen is a second-year Master's student at the School of Mathematical Sciences, Hebei Normal University. Since May 2024, she has been conducting research on power semigroups under the guidance of Salvatore Tringali. Together with her supervisor, she has completed two papers in this area, both of which are currently under review. In September 2025, she was one of the organizers of the 1st International Workshop on Power Semigroups at the School of Mathematical Sciences of Hebei Normal University (details here) and a contributed speaker at the International Conference on Additive Combinatorics held at the Zhuhai campus of Sun Yat-sen University (details here).

References

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